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Analysis of Fractional Digital Burnout Equation: Approximate Solutions and the Effect of the Fractional Order

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Abstract:
Fractional calculus provides powerful tools for modeling memory-dependent systems and complex real-world problems in mathematical biology and biomedical models. In this paper, the fractional digital burnout equation is studied, where we used the Homotopy Perturbation Method (HPM) coupled with the Laplace transform to obtain approximate analytical solutions. The fractional derivative is considered in the Caputo sense. Particular attention is devoted to investigating the effect of the fractional-order parameter α on the behavior of the solution. Some illustrative examples are presented, and graphical representations of the approximate solutions for different values of α are provided to explain the effect of the fractional order on the model dynamics and solution behavior.
Keywords: Caputo fractional derivative, Digital burnout models, Homotopy perturbation method, Laplace transform.

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تحليل معادلة الإحتراق الكسرية: الحلول التقريبية وتأثير الرتبة الكسرية

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الملخص

حساب التفاضل والتكامل الكسري يُوفرلنا أدوات فعّالة لنمذجة الأنظمة المعتمدة على الذاكرة والمشكلات المعقدة في العالم الحقيقي في علم الأحياء الرياضي والنماذج الطبية الحيوية. في هذه الورقة قدمنا معادلة الاحتراق الرقمي من الرتبة الكسرية، حيث استخدمنا طريقة اضطراب التماثل (HPM) المقترنة بتحويل لابلاس للحصول على حلول تحليلية تقريبية. هنا اعتبرنا المشتق الكسري وفقًا لمفهوم كابوتو. سوف نعطي اهتمامًا خاصًا لدراسة تأثير معامل الرتبة الكسرية α على سلوك الحل حيث قدمنا العديد من الأمثلة التوضيحية، مع رسوم بيانية للحلول التقريبية لقيم مختلفة لـ α لتوضيح تأثير الرتبة الكسرية على ديناميكيات النموذج وعلى سلوك الحل.

الكلمات المفتاحية: تفاضل كابوتو الكسري، الإحتراق الرقمي، طريقة الاضطراب، تحويل لابلاس.

1. Introduction.

Fractional calculus has attracted much attention recently. This is mostly due to the fact that it provides an efficient and excellent instrument for describing many dynamical phenomena arising in engineering and other scientific disciplines such as physics, chemistry, biology, control theory, and medical sciences ([1],[2],[3],[4],[5],[6],[7],[8],[9]).

The Caputo fractional derivative is considered as an effective tool for modeling real-world phenomena; for example, Podlubny in [2] introduced the fundamental theory of fractional differential equations and their applications in dynamic systems. On the same side, Diethelm [5] presented the analytical properties of Caputo fractional models and their ability to represent nonlinear systems with memory effects.

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Nowadays, solving differential equations of fractional order is considered one of the most important fields in fractional calculus. Numerous analytical and numerical methods have been developed to obtain approximate or exact solutions of this type of equations, for example Laplace transform ([10], [11], [12]), Adomian decomposition method ([13], [14], [15]), variational iteration method (VIM) ([16], [17]), and homotopy perturbation method ([18], [19], [20], [21], [22], [23]).
Mathematical modeling plays an important role in understanding the dynamic behavior of a large number of psychological and behavioral problems [24]. Classical differential equations have been widely used to describe many real-life systems, but they sometimes fail to describe the memory effects. Therefore, scientists and researchers replaced the classical derivatives with fractional derivatives which provide a more realistic description of the effect of the basic states on the present dynamic.
Burnout is a state of physical, psychological, or emotional strain resulting from a prolonged and continuous exposure to stress, and affects overall performance and productivity. There are many types of burnout such as, Occupational Burnout, Academic Burnout, Emotional Burnout, Digital Burnout, etc. These types can be modeled using differential equations to study them. The most important type nowadays is Digital Burnout that will be modeled and discussed here.
If we consider the digital burnout and psychological stress variables as a continuous function on the time, then we can establish the relationship between these functions by differential equation that describes and analyzes them with initial conditions. So, we can get the model described using ordinary differential equations, where the most of real-life situations digital burnout depends on collecting of previous stress, continuous tiring and exposure to digital environments due to this reason, fractional order model more suitable and accurate to these phenomena.
In this paper, suppose that $u(t)$ represents the level of digital burnout at time t, and $D(t)$ denotes the digital usage intensity and $R(t)$ represents the recovery factor (rest - sleep - ….). The burnout rate can be written as:

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$$u'(\mathrm{t}) = \rho D(t)u(t) - \lambda R(t) - \mu u^2(t) \;;\quad u(0) = u_0 \qquad (1.1)$$

where $\lambda, \mu > 0$, ρ is the effect of digital usage λ recovery , and μ is the natural reduction of burnout .

If we suppose that the burnout depends on the past experiments and the accumulated stress, then it is suitable to replace the classical derivative by fractional derivative, so we get the fractional digital burnout equation on the form:

$$D_t^{\alpha} u(t) = \rho D(t)u(t) - \lambda R(t) - \mu u^2(t), \qquad 0 < \alpha \leq 1 \qquad (1.2)$$

with initial condition

$$u(0) = u_0 \qquad (1.3)$$

where D_t^{α} is the fractional derivative in Caputo sense.

This equation can be considered as a special case of fractional Riccati differential equation. Several, factors that affect the digital burnout model, such as values of α, as well as the functional expression of the nonhomogeneous term $\mathrm{R(t)}$, in this work, we focus on the effect of α.

In this paper we have coupled the homotopy perturbation method with Laplace transform method (HPLTM) and used it to obtain analytical approximate solution of fractional digital burnout equation (linear-and nonlinear), and analyze the effect of the fractional order on the solution behavior.

The reminder of this paper is organized as follows. In section 2 we recalled the preliminary definitions and theories in fractional calculus which we need them in this paper. In section 3 we introduced the basic idea of homotopy perturbation method. In section 4 we described the solution algorithm of fractional digital burnout equation using (HPLTM). In section 5 we gave special cases of the proposed equations and solved by proposed method and discuss the results. Finally in section 6, we gave the concluding remarks.

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2. Preliminaries

In this section, we present some definitions and properties of fractional calculus
that will be useful in this work

Definition2.1. The Riemann-Liouville fractional integral is defined as **[2]**

$$J^{\alpha}[f(x)] = \frac{1}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1} f(t)\,dt, \quad \alpha > 0, x > 0 \quad (2.1)$$

In particular $J^0\,[f(x)] = f(x)$,

For Riemann-Liouville fractional integral, one has

$$J^{\alpha}x^{\gamma} = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)}\,x^{\gamma+\alpha} \qquad (2.2)$$

Definition 2.2. The Caputo fractional derivative is defined as [2]

$$D_x^{\alpha}[f(x)] = {}^{m-\alpha}_{J}D^m f(x) = \frac{1}{\Gamma(\alpha)}\int_0^x (x-t)^{m-\alpha-1} f^m(t)\,dt \quad (2.3)$$

For $m-1 < \alpha \le m$, $m \in N, x > 0$, $f \in C_{\mu}^{m}$, $\mu \ge 1$

Lemma 2.1. [2]

If $m-1 < \alpha \le m$, $m \in N$, $f \in C_{\mu}^{m}$, $\mu \ge -1$

Then the following two properties hold.

1- $D^{\alpha}\left[{}^{\alpha}_{J}f(x)\right] = f(x)$

2- $J^{\alpha}[D^{\alpha}f(x)] = f(x) - \sum_{k=1}^{m-1} F^k(0)\frac{x^k}{k!}$

Definition2.3. The Mittag–Leffler function $E_{\alpha}(x)$ with $\alpha > 0$ is defined by the following series representation, valid in the whole complex plane [25].

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$$E_\alpha(x) = \sum_{n=0}^{\infty} \frac{x^n}{\Gamma(\alpha n+1)} \quad (2.4)$$

The Mittag-Leffler function with two parameters $E_{\alpha,\beta}(x)$ defined as follows **[25]**

$$E_{\alpha,\beta}(x) = \sum \frac{x^n}{\Gamma(\alpha n+\beta)}_{n=0}^{\infty}, \quad \alpha,\beta > 0 \quad (2.5)$$

Its *k*th derivative is given by

$$E_{\alpha,\beta}^{(k)}(x) = \sum_{n=0}^{\infty} \frac{(n+k)!\ x^n}{n!\ \Gamma(\alpha n+\alpha k+\beta)}, k = 0,1,2,3, \dots \quad (2.6)$$

We find it convenient to introduce the function

$$\sigma_k(t, a, \alpha, \beta) = t^{\alpha k+\beta-1} E_{\alpha,\beta}^{(k)}(\pm a t^\alpha) \quad (2.7)$$

Definition 2.4. The Laplace transform of a function $f(x), x \geq 0$ denoted by F(s), is defined by [26]

$$\mathcal{L}[f(x)] = \mathrm{F}(s) = \int_0^\infty e^{-sx} f(x)\, dx \quad (2.8)$$

where s is the transform parameter, which is assumed to be a real variable.
Theorem 2.1. For any two functions $f(t)$and $g(t)$with Laplace transforms $F(s)$and $G(s)$ we have [26]

$$\mathcal{L}(f * g) = F(s) \cdot G(s).$$

Similarly, if $\mathcal{L}^{-1}[F(s)] = f(t)$ and $\mathcal{L}^{-1}[G(s)] = g(t)$ then,
$\mathcal{L}^{-1}[F(s) \cdot G(s)] = f(t) * g(t)$

Definition 2.5. The Laplace transform of the Caputo fractional derivative is defined as [2]

$$\mathcal{L}[D^\alpha f(x)\,] = s^\alpha F(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1} f^n(0) \quad (2.9)$$

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Laplace transform of $\sigma_k(t, a, \alpha, \beta)$ was evaluated by Podlubny [2]

$$\mathcal{L}[\sigma_k(t,a,\alpha,\beta)] = \frac{k!\ s^{\alpha-\beta}}{(s^\alpha \pm a)^{k+1}},\quad Re(s) > |a|^{\frac{1}{\alpha}} \qquad (2.10)$$

Hence

$$\mathcal{L}^{-1}\left[\frac{k!\ s^{\alpha-\beta}}{(s^\alpha \pm a)^{k+1}}\right] = t^{\alpha k+\beta-1}E_{\alpha,\beta}^{(k)}(\pm a t^\alpha),\quad Re(s) > |a|^{\frac{1}{\alpha}} \quad (2.11)$$

3. Homotopy perturbation method

In this section, we introduce the homotopy perturbation method for general type of the nonlinear differential equation with boundary conditions

$$A(u) - f(r) = 0,\quad r \in \mathbb{R} \qquad (3.1)$$

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0\ ,\ r \in \Gamma \qquad (3.2)$$

where B is a boundary operator and A is a general differential operator and can be divided into parts linear operator L and a nonlinear operator N.

Therefore, eq. (3.1) can be rewritten as follows:

$$L(u) + N(u) - f(r) = 0 \qquad (3.3)$$

By the homotopy technique ([18], [19]), we define a homotopy as follows:

$$\mathrm{H}(\mathrm{r},\mathrm{p}) :\ \Omega \times [0,1] \longrightarrow R$$

$$\mathrm{H}(\mathrm{u},\mathrm{p}) = (1-\mathrm{p})[L(u) - L(u_0)] + p[A(u) - f(r)] = 0, p \in [0,1], r \in \Omega \quad (3.4)$$

Or

$$\mathrm{H}(\mathrm{u},\mathrm{p}) = L(u) - L(u_0) + pL(u_0) + p[N(u) - f(r)] = 0,\ p \in [0,1] (3.5)$$

where $p \in [0,1]$ is an embedding parameter and u_0 is an initial approximation of eq. (3.1) which satisfies the boundary conditions. From (3.4) and (3.5) we have

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$$\begin{aligned} H(u,0) &= L(u) - L(u_0) = 0 \\ H(u,1) &= A(u) - f(r) = 0 \end{aligned} \tag{3.6}$$

As the embedding parameter of p varies continuously of p from zero to unity is just that of (r,p) from $u_0(r)$ to $u(r)$. In topology this is called deformation and $L(u) - L(u_0)$ and $A(u) - f(r)$ are called homotopic.

Now, assume the solution of equations (3.4) and (3.5) can be expressed as a power series in p, that is

$$\vartheta = \sum_{k=0}^{\infty} p^k u_k \tag{3.7}$$

Substituting (3.7) in (3.6) and comparing the coefficients of power of p yields a successive procedure to determine u_k .

Finally, the approximate solution of eq. (3.1) can be obtained by setting $p = 1$ in eq. (3.7)

4. Homotopy Perturbation Method Coupled with Laplace Transform

To illustrate the basic idea of this method, we consider the following fractional nonlinear Digital Burnout equation with initial condition in the form

$$c_{D_t^\alpha} u(t) = \lambda D(t) - \mu u(t) - \eta u^2(t) + \rho D(t)u(t), 0 < \alpha \leq 1 \tag{4.1}$$

Firstly, applying the Laplace transform of both sides of (4.1), we have

$$\mathcal{L}[c_{D_t^\alpha} u(t)] = \mathcal{L}[\lambda D(t)] - \mathcal{L}[\mu u(t)] + \mathcal{L}[\rho D(t)u(t)] - \mathcal{L}[\eta u^2(t)] \tag{4.2}$$

Using the differential property of Laplace transform, we get

$$\mathcal{L}[u(t)] = \frac{u_0}{s} + s^{-\alpha}\mathcal{L}[\lambda D(t)] - s^{-\alpha}\mathcal{L}[\mu u(t)] + s^{-\alpha}\mathcal{L}[\rho D(t)u(t)] - s^{-\alpha}\mathcal{L}[\eta u^2(t)] \tag{4.3}$$

Operating the inverse Laplace transform on both sides of (4.3), we have

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$$u(t) = M(t) + \mathcal{L}^{-1}(-s^{-\alpha}\mathcal{L}[\mu u(t)] + s^{-\alpha}\mathcal{L}[\rho D(t)u(t)] -s^{-\alpha}\mathcal{L}[\eta u^2(t)]) \qquad (4.4)$$

where $M(t)$ represents the term arising the source term and the prescribed initial condition.

Now, applying (HPM), we get

$$\sum_{n=0}^{\infty} p^n u_n(\text{t}) = M(t) + P\{\mathcal{L}^{-1}(s^{-\alpha}\mathcal{L}[\sum_{n=0}^{\infty} P^n H_n(\text{u})] - s^{-\alpha}\mathcal{L}[\eta \sum_{n=0}^{\infty} P^n N_n(\text{u})])\} \qquad (4.5)$$

where $H_n(\text{u}) = -\mu u(t) + \rho D(t)u(t)$ and $N_n(\text{u})$ is He's polynomials.

He's polynomials for u^2 can be calculated as follows

$$N_0(\text{u}) = u_0^2$$
$$N_1(\text{u}) = 2u_0u_1 \qquad (4.6)$$
$$N_2(\text{u}) = u_1^2 + 2u_0u_2$$

Equating the corresponding power of p on both sides in equation(4.5), we have

$$p^0:\ u_0(t) = M(t)$$
$$p^1:\ u_1(t) = \mathcal{L}^{-1}s^{-\alpha}\mathcal{L}[H_0(\text{u})] - s^{-\alpha}\mathcal{L}[\eta N_0(\text{u})] \qquad (4.7)$$
$$p^2:\ u_2(t) = \mathcal{L}^{-1}s^{-\alpha}\mathcal{L}[H_1(\text{u})] - s^{-\alpha}\mathcal{L}[\eta N_1(\text{u})]$$

Proceeding in the same manner, the rest of the components $u_n(t)$ can be completely obtained, and the series solution is thus entirely determined. Finally, we approximate the solution $u(t)$ by truncated series

$$u(t) = \lim_{n\to\infty} \sum_{n=0}^{N} u_n(t) \qquad (4.8)$$

This series solutions generally converge very rapidly.

5. Applications

In this section, we discuss the implementation of proposed method for several case of digital Burnout model.

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Example5.1. Consider the following linear fractional Digital Burnout equation with initial condition

$$D^{\alpha}u(t) = \mathrm{t} - u(\mathrm{t}) \qquad ,0 < \alpha \leq 1\,,\;\; u(0) = 0 \tag{5.1}$$

with the exact solution

$$u(t) = 1 - e^{-t} \tag{5.2}$$

where t is burnout source and $-u(\mathrm{t})$ is recovery term.
Applying Laplace transform on both sides on eq. (5.1). subject to initial condition (5.2), we get

$$\mathcal{L}\{\mathrm{u}(t)\} = \frac{1}{s^{2+\alpha}} - s^{-\alpha}\mathcal{L}\{\mathrm{u}(t)\} \tag{5.3}$$

Operating the inverse of Laplace transform on both sides in (5.3)we have

$$u(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha+2}}\right\} - \mathcal{L}^{-1}\{s^{-\alpha}\,\mathcal{L}\{\mathrm{u}(t)\}\} \tag{5.4}$$

Now, applying homotopy perturbation method

$$\sum_{n=0}^{\infty} p^n u_n(t) = \frac{t^{\alpha+1}}{\Gamma(\alpha+2)} - p\mathcal{L}^{-1}[s^{-\alpha}\,\mathcal{L}\{u_n(t)\}] \tag{5.5}$$

$$\begin{aligned}
p^0&: u_0(t) = \frac{t^{\alpha+1}}{\Gamma(\alpha+2)}\\
p^1&: u_1(t) = -\mathcal{L}^{-1}[s^{-\alpha}\,\mathcal{L}\{u_0(t)\}]\\
p^2&: u_2(t) = -\mathcal{L}^{-1}[s^{-\alpha}\,\mathcal{L}\{u_1(t)\}]\\
p^3&: u_3(t) = -\mathcal{L}^{-1}[s^{-\alpha}\,\mathcal{L}\{u_2(t)\}]\\
&\vdots\\
p^n&: u_n(t) = -\mathcal{L}^{-1}[s^{-\alpha}\,\mathcal{L}\{u_{n-1}(t)\}]
\end{aligned} \tag{5.6}$$

So, the solution $u(t)$ the problem is given by

$$\begin{aligned}
u(t) &= \lim_{n\to\infty} \sum_{n=0}^{N} u_n(t)\\
&= \frac{t^{\alpha+1}}{\Gamma(\alpha+2)} - \frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} + \frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)} - \frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)} + \cdots
\end{aligned}$$

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$$-\frac{t^{n\alpha+1}}{\Gamma(n\alpha+2)} \qquad (5.7)$$

which is an exact solution of the given fractional Digital Burnout Equation(5.1) for $\alpha = 1$.

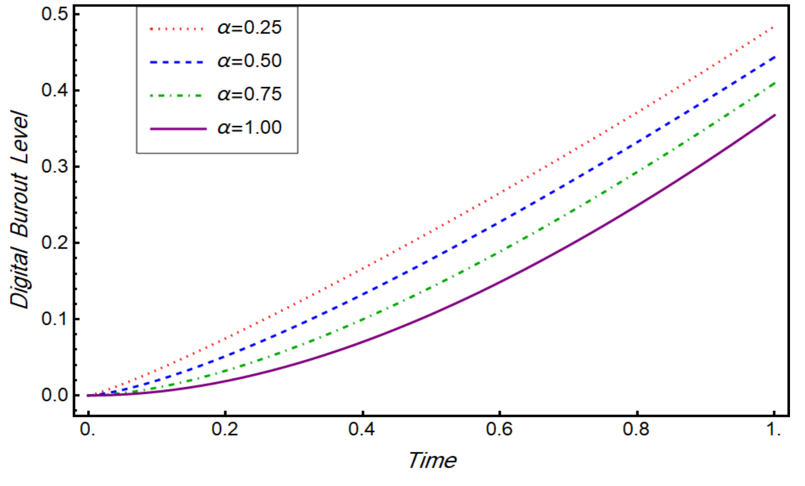

Figure 1: Approximate solution for equation (5.1) for different values of α

Example 5.2: We consider the following linear fractional Digital Burnout equation with initial condition

$$D^{\alpha}(u) = e^{t} - \mathrm{u}(t) \qquad u(0) = 0 \qquad (5.8)$$

with the exact solution

$$u(t) = \sinh(t) \qquad (5.9)$$

Applying Laplace transform on both sides on eq. (5.8) subject to initial condition(5.9), we get

$$\mathcal{L}\{\mathrm{u}(t)\} = \frac{1}{s^{\alpha}(s-1)} - s^{-\alpha}\mathcal{L}\{\mathrm{u}(t)\} \qquad (5.10)$$

Operating the inverse of Laplace transform on both sides in (5.10), we have

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$$u(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha}(s-1)}\right\} - \mathcal{L}^{-1}\left\{s^{-\alpha}\mathcal{L}\{u(t)\}\right\} \tag{5.11}$$

Now, applying homotopy perturbation method

$$\sum_{n=0}^{\infty} p^n u_n(t) = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}(s-1)}\right] - p\mathcal{L}^{-1}\left[s^{-\alpha}\mathcal{L}\{u_n(t)\}\right] \tag{5.12}$$

$$\begin{aligned}
&p^0: u_0(t) = t^{\alpha}E_{1,\alpha+1}(t)\\
&p^1: u_1(t) = -\mathcal{L}^{-1}[s^{-\alpha}\,\mathcal{L}\{u_0(t)\}]\\
&p^2: u_2(t) = -\mathcal{L}^{-1}[s^{-\alpha}\,\mathcal{L}\{u_1(t)\}]\\
&p^3: u_3(t) = -\mathcal{L}^{-1}[s^{-\alpha}\,\mathcal{L}\{u_2(t)\}]\\
&\quad\vdots\\
&p^n: u_n(t) = -\mathcal{L}^{-1}[s^{-\alpha}\,\mathcal{L}\{u_{n-1}(t)\}]
\end{aligned} \tag{5.13}$$

So, the solution $u(t)$ the problem is given by

$$\begin{aligned}
u(t) &= \lim_{n\to\infty} \textstyle\sum_{n=0}^{N} u_n(t)\\
u(t) &= t^{\alpha}E_{1,\alpha+1}(t) - t^{2\alpha}E_{1,1+2\alpha}(t) + t^{3\alpha}E_{1,1+3\alpha}(t)\\
&\quad - t^{4\alpha}E_{1,1+4\alpha}(t) + \cdots - t^{n\alpha}E_{1,1+n\alpha}(t)
\end{aligned} \tag{5.14}$$

which is an exact solution of the given fractional Digital Burnout Equation(5.8) for $\alpha = 1$.

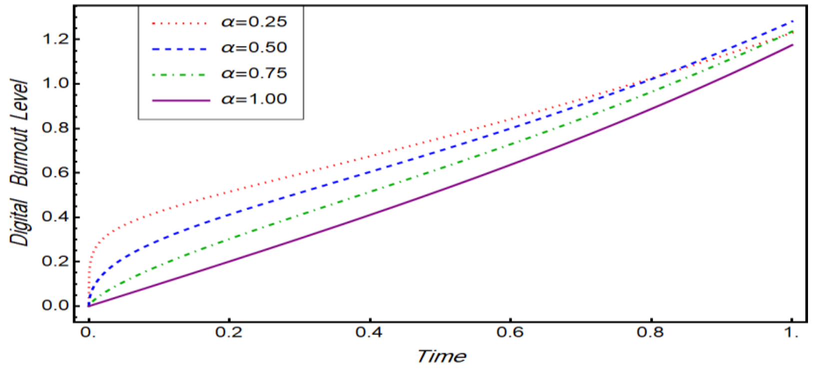

Figure 2: Approximate solution for equation (5.8) for different values of α

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Example5.3. We consider the following nonlinear fractional Digital Burnout equation with initial condition

$$D^{\alpha}(u) = e^{t} - \mathrm{u}(t) - \mathrm{u}^2(t), \qquad 0 < \alpha \le 1 \tag{5.15}$$

Subject to initial condition

$$u(0) = 0 \tag{5.16}$$

Applying Laplace transform on both sides of (5.15)with initial condition(5.16), we get

$$\mathcal{L}\{D^{\alpha}(u)\} = \mathcal{L}\{e^{t}\} - \mathcal{L}\{\mathrm{u}(t)\} - \mathcal{L}\{\mathrm{u}^2(t)\} \tag{5.17}$$

Operating the inverse of Laplace transform on both sides in(5.17), we have

$$\mathrm{u}(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha}(s-1)}\right\} - \mathcal{L}^{-1}\left[s^{-\alpha}\mathcal{L}\{\mathrm{u}(t)\} - s^{-\alpha}\,\mathcal{L}\{\mathrm{u}^2(t)\}\right] \tag{5.18}$$

Now, applying homotopy perturbation method

$$\sum_{n=0}^{\infty} p^n u_n(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha}(s-1)}\right\} - \mathrm{p}\mathcal{L}^{-1}\left[s^{-\alpha}\mathcal{L}\{\mathrm{u}(t)\} - s^{-\alpha}\,\mathcal{L}\{\mathrm{u}^2(t)\}\right] \tag{5.19}$$

$$\begin{aligned}
u_0 &= \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha}(s-1)}\right\} \\
&= t^{\alpha}E_{1,1+\alpha}(t) \\
u_1 &= \mathcal{L}^{-1}\left[-s^{-\alpha}\mathcal{L}\{u_0(t)\} - s^{-\alpha}\,\mathcal{L}\{u_0{}^2(t)\}\right] \\
&= -t^{2\alpha}E_{1,1+2\alpha}(t) - t^{1+3\alpha}E^{(1)}{}_{1,1+3\alpha}(t) \\
u_2 &= \mathcal{L}^{-1}\left[-s^{-\alpha}\mathcal{L}\{u_1(t)\} - s^{-\alpha}\,\mathcal{L}\{2u_0u_1\}\right] \\
&= -t^{3\alpha}E_{1,1+3\alpha}(t) - t^{1+4\alpha}E^{(1)}{}_{1,1+4\alpha}(t) - 2t^{1+4\alpha}E^{(1)}{}_{1,1+4\alpha}(t) \\
&\quad +2\,t^{1+5\alpha}E^{(2)}{}_{1,1+5\alpha}(t) \qquad (5.20) \\
u_3 &= t^{4\alpha}E_{1,1+4\alpha}(t) + t^{1+5\alpha}E^{(1)}{}_{1,1+5\alpha}(t) + t^{2+6\alpha}E^{(2)}{}_{1,1+6\alpha}(t) \\
&\quad + \frac{1}{2}\,t^{3+7\alpha}E^{(3)}{}_{1,1+7\alpha}(t) \\
&\vdots
\end{aligned}$$

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The approximate solution of above equation is

$$u_n(t) = t^{\alpha}E_{1,1+\alpha}(t) - t^{2\alpha}E_{1,1+2\alpha}(t) - t^{3\alpha+1}E^{(1)}{}_{1,1+3\alpha}(t) - t^{3\alpha}E_{1,1+3\alpha}(t) - 3t^{4\alpha+1}E^{(1)}{}_{1,1+4\alpha}(t) + 2\,t^{1+5\alpha}E^{(2)}{}_{1,1+5\alpha}(t) + t^{4\alpha}E_{1,1+4\alpha}(t) + t^{1+5\alpha}E^{(1)}{}_{1,1+5\alpha}(t) + t^{2+6\alpha}E^{(2)}{}_{1,1+6\alpha}(t) + \frac{1}{2}\,t^{3+7\alpha}E^{(3)}{}_{1,1+7\alpha}(t) \ldots \quad (5.21)$$

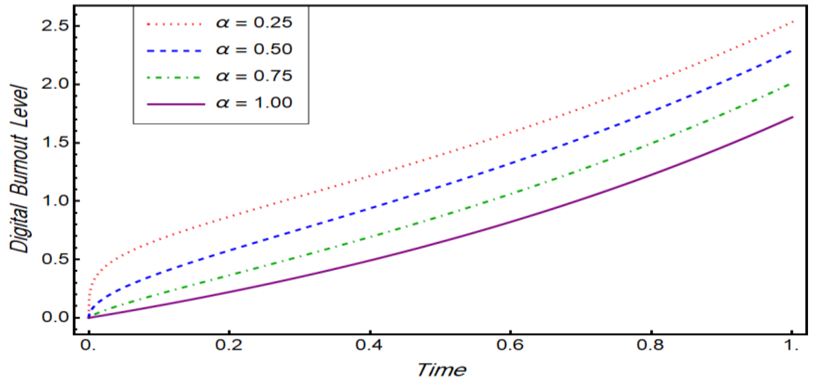

Figure 3: Approximate solution for equation (5.15)-(5.16) for different values of α

6. Result Discussion and Conclusion

Figures (1), (2), and (3) show the behavior of the solution at different values of α (α=0.25, α=0.50, α=0.75, α=1). In all figures, we see that u(t) (representing the level of digital burnout) increases with time for all values of the fractional order parameter α. In particular, at α=0.25, u(t) reaches the highest values, whereas at α=1, it gives the lowest values over the considered time interval. This indicates that the fractional order α has a significant influence on the dynamics of the model. From another point of view, decreasing the fractional order increases the memory effect of the system.

In Conclusion, the obtained results demonstrate that fractional differential equations provide an effective framework for modeling digital burnout. In this article, we used a fractional-order digital

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burnout Model (1.2)-(1.3) for investigating the memory effect resulting from the continuous use of digital technologies, where the current burnout level depends not only on the present state but is also influenced by previous exposure to digital stressors. Therefore, the fractional order model provides a more realistic representation of digital burnout than the classical order solution $\alpha=1$ over time. The numerical results further reveal that burnout evolves more gradually than compared with the classical model due to this behavior is attributed to the memory effect of the fractional derivative, which reflects the influence of fractional dynamics

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